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On the Simulation of Harmonically Related Signals

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Narrowband harmonically related signals, embedded in broadband noise and other unrelated signals, occur in specific applications. To study these signals and to evaluate techniques for their discernment, it is convenient to model or simulate harmonically related signals in the laboratory. A means of accomplishing this task on a conventional digital computer is developed in discrete algorithm format. The resulting simulator uses parameters that allow one to select the mean frequency and harmonic ratio of the signals and to control both the extent and autocorrelation (or power spectral density) of the random signal-frequency fluctuations. Broadband noise and other nonharmonically related signals may also be accommodated in the simlator concept. Examples, using the algorithm, demonstrate its performance and its conformance with theoretical predictions.

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CONTENTS

ON THE SIMULATION OF HARMONICALLY RELATED SIGNALS

INTRODUCTION

In a number of applications, narrowband signal components occur that are harmonically related. To understand harmonically related signals, it is sufficient to recognize that over any observation (or analysis) interval, a narrowband signal $u(t)$ may be represented as $A(t)\exp\{i\psi(t)\}$, where $A(t) \geq 0$ is the amplitude function and $\psi(t)$ is the phase function of the relevant signal. Any two narrowband signals $u_p(t)$ and $u_q(t)$ are considered to be harmonically related when the phase functions of the two signals are linearly related. That is, when

$$\psi_q(t) = q\psi_p(t) + \text{ a constant} \tag{1a}$$

and the harmonic ratio q is any real number (not necessarily an integer). The instantaneous frequencies of the two functions are therefore related as

$$f_q(t) = \dot{\psi}_q(t)/2\pi = \bar{f}_q + \nu_q(t)$$

$$= q\dot{\psi}_p(t)/2\pi = q[\bar{f}_p + \nu_p(t)] = qf_p(t), \tag{1b}$$

where the "dot" over the variable implies the time derivative. Here $f_p(t)$, $f_q(t)$ are the instantaneous frequencies of the two signals, $\bar{f}_p$, $\bar{f}_q$ are the mean values of the instantaneous frequencies, and $\nu_p(t)$, $\nu_q(t)$ are the zero-mean fluctuating components. For convenience, it is assumed that $f_p(t) < f_q(t)$ and $1 < q$.

This report formulates discrete mathematical functions that simulate the harmonic relations given in Eq. (1) for implementation on a digital computer. The fluctuating components are random but bounded in their excursion, and the fluctuation dynamics (rate of change) are controlled to emulate signals that occur in practice.

FORMULATION OF THE PHASE FUNCTIONS

To formulate the discrete phase functions of harmonically related signals sampled at a rate of f_s Hz, let

$$\psi_n = \psi(n\Delta t) = 2\pi Z'_n,$$

where the sample-time increment Δt is equal to $1/f_s$ and Z'_n is the integral of the instantaneous frequency f_n. That is,

$$Z'_n = \sum_{j=0}^{n} f_j \Delta t = \sum_{j=0}^{n} \frac{f_j}{f_s}.$$

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This expression may be written in recurrence form as

$$Z'_n = Z'_{n-1} + \frac{f_n}{f_s},$$

where $Z'_{-1} = 0$. Since phase need be determined only to modulo 2π (integral of instantaneous frequency to modulo 1), and since the modulo of a sum of terms is equal to the modulo of the sum of the modulos of each term, let

$$Z_n = Z'_n \text{ modulo } 1 = \left[Z_{n-1} + \frac{f_n}{f_s} \right] \text{ modulo } 1 .$$

Applying the recurrence formula to the harmonically related instantaneous frequencies given in Eq. (1b) and using the parameters for the q-subscripted (or higher frequency) signal, gives

$$Z_{q,n} = \mathrm{Frac}\{Z_{q,n-1} + (\bar{f}_q + \nu_n)/f_s\} \tag{2a}$$

and

$$Z_{p,n} = \mathrm{Frac}\{Z_{p,n-1} + (\bar{f}_q + \nu_n)/qf_s\}. \tag{2b}$$

Here $Z_{p,-1} = Z_{q,-1} = 0$, and the function $\mathrm{Frac}\{\ldots\}$ implies taking only the fractional part of the decimal argument. It is recognized that $\bar{f}_q = q\bar{f}_p$ is the mean frequency of the higher frequency signal and $\nu_n = \nu_q(n\Delta t) = q\,\nu_p(n\Delta t)$ is the fluctuating signal component of the higher frequency signal. The resulting modulo-phase functions of the two harmonically related signals are

$$\psi_{p,n} = 2\pi Z_{p,n} \text{ and } \psi_{q,n} = 2\pi Z_{q,n} . \tag{2c}$$

FORMULATION OF THE FREQUENCY-FLUCTUATION FUNCTION

At this point, a particular zero-mean deterministic function for ν_n may be chosen. In this report, however, the frequency function is chosen to be truly random and Gaussian-like but bounded in its peak excursion from zero. To accomplish this, let ζ_n be a zero-mean Gaussian statistic with standard deviation σ_ζ Hz. To bound the frequency-fluctuation function to absolute values no greater than $\alpha\,\sigma_\zeta$, let

$$\nu_n = \alpha\,\sigma_\zeta \,\mathrm{Frac}\,\{\zeta_n/\alpha\,\sigma_\zeta\}. \tag{3a}$$

where $\mathrm{Frac}\{\cdots\}$ implies the fractional part of the argument while retaining the sign of ζ_n. The statistic ν_n is therefore a zero-mean random number whose probability density is

$$p_\nu(\nu) = \begin{cases} \sum_{j=0}^{\infty} p_\zeta(|\nu| + j\alpha\,\sigma_\zeta) & \text{for } |\nu| < \alpha\,\sigma_\zeta \\ 0 & \text{for } \alpha\,\sigma_\zeta \le |\nu| \end{cases} \tag{3b}$$

and

$$p_{\zeta}(x) = \frac{1}{\sqrt{2\pi}\sigma_{\zeta}} e^{-\frac{x^2}{2\sigma_{\zeta}^2}} .$$

The probability that $-\nu_o \leq \nu \leq \nu_o$ for $0 < \nu_o < \alpha\,\sigma_{\zeta}$ is

$$P(|\nu| < \nu_0) = 2 \sum_{j=0}^{\infty} [N\{\nu_0/\sigma_{\zeta} + j\alpha\} - N\{j\alpha\}], \tag{3c}$$

where $N\{y\}$ is the normal or Gaussian distribution function

$$N\{y\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{x^2}{2}} dx .$$

FORMULATION OF THE GAUSSIAN STATISTIC ζ_n

To emulate the frequency fluctuations of real signals, the rate of variation (autocorrelation or power spectral density) of the statistic ζ_n must be controlled. This may be accomplished by low-pass filtering a running sequence of independent random samples. Consider, then, that ξ_n is a zero-mean sequence of independent random samples with standard deviation σ_{ζ}. The desired output ζ_n can be obtained by processing ξ_n through the discrete equivalence of a single-pole infinite-impulse response (IIR) filter [1,2], whose normalized unit-impulse response is

$$h_n = (1 - \beta)\beta^n f_s , \tag{4}$$

where $f_s = 1/\Delta t$ is the sample rate and $\beta < 1$ is the filter design parameter that controls the effective smoothing time or noise bandwidth of the filter. The effective bandwidth of the discrete filter is

$$B = \frac{1}{2} \sum_{n=0}^{\infty} h_n^2 \Delta t = \frac{1-\beta}{1+\beta} \frac{f_s}{2} , \tag{5a}$$

and the effective smoothing time is

$$\tau = \frac{1}{2B} = \frac{1+\beta}{1-\beta} \frac{1}{f_s} . \tag{5b}$$

The normalized filter autocorrelation function is, from Eqs. (4) and (5b),

$$\gamma_n = \tau \sum_{j=0}^{\infty} h_j h_{j+|n|} \Delta t = \beta^{|n|} = \beta^{f_s|t|} \tag{5c}$$

In terms of the smoothing time τ, the bandwidth B, or the autocorrelation at a time delay t ($n = t/\Delta t = f_s t$), the filter parameter becomes

$$\beta = \frac{f_s\tau - 1}{f_s\tau + 1} = \frac{f_s - 2B}{f_s + 2B} = \gamma_n^{1/f_s t}. \tag{6a}$$

The analog equivalent of this discrete filter is a single-pole low-pass filter whose cutoff frequency (3 dB down point) is

$$f_c = \frac{1}{\pi\tau} = \frac{2B}{\pi} = \frac{1-\beta}{1+\beta}\frac{f_s}{\pi}. \tag{6b}$$

The effective bandwidth of the random statistic ξ_n, sampled at a rate f_s, is the Nyquist rate $f_s/2$. Consequently, the variance of ξ_n at the output of the filter is reduced by the factor $2B/f_s = 1/f_s\tau$. To make the standard deviation at the filter output equal to the desired standard deviation σ_ζ, the random statistic ξ_n must be multiplied by the factor c where, from Eq. (5b),

$$c = \sqrt{f_s\tau}\,\frac{\sigma_\zeta}{\sigma_\xi} = \sqrt{\frac{1+\beta}{1-\beta}}\,\frac{\sigma_\zeta}{\sigma_\xi}. \tag{6c}$$

The output of the filter is obtained by using the discrete convolution relation [1,2]

$$\zeta_n = \sum_{j=0}^{\infty} h_j c\,\xi_{n-j}\Delta t. \tag{7a}$$

From Eqs. (4), (6c) and (7a), the Gaussian statistic ζ_n is formulated as

$$\zeta_n = (1-\beta)\sum_{j=0}^{\infty}\beta^j c\,\xi_{n-j} = \beta\zeta_{n-1} + (1-\beta)c\,\xi_n$$

$$= \beta\zeta_{n-1} + \sqrt{1-\beta^2}\,\frac{\sigma_\zeta}{\sigma_\xi}\xi_n. \tag{7b}$$

The random sequence ξ_n need not be Gaussian for ζ_n to be Gaussian as long as $f_s\tau = (1+\beta)/(1-\beta)$ is large compared with one (by reason of the central-limit theorem). The value $f_s\tau$ is the effective number of independent samples of ξ_n in the filter smoothing process. This value is expected to be very large in practical applications.

FORMULATION OF THE RANDOM SEQUENCE ξ_n

A convenient method of generating the random sequence ξ_n on the digital computer is to randomly select digital values R_n between 0 and 1 at the sample rate f_s and to formulate ξ_n as

$$\xi_n = R_n - 0.5, \tag{8a}$$

where it is assumed that all values of R_n are equally likely to occur with each sample and thus are independent. The random statistic has zero mean, and for M discrete values between 0 and 1 the standard deviation of ξ_n is

$$\sigma_\xi = \sqrt{\frac{M+1}{12(M-1)}}. \tag{8b}$$

For M large, the standard deviation may be approximated as $1/\sqrt{12}$.

SIMULATOR DESIGN EQUATIONS

The design relations to simulate two narrowband random signals embedded in random noise is now given in an orderly manner for convenient use in applications. The combined signal is considered to be a discrete signal sampled at a rate f_s and expressed in the form

$$s_n = a_p \sin(2\pi Z_{p,n}) + a_q \sin(2\pi Z_{q,n}) + \eta_n , \tag{9}$$

where η_n is an independent zero-mean Gaussian statistic with variance σ_η^2, and the functions $2\pi Z_{p,n}$ and $2\pi Z_{q,n}$ are harmonically related random phase functions (modulo 2π).

Input Parameters

The following parameters are to be entered into the simulator program.

f_s	is the signal sample rate in Hz,
$q > 1$	is the harmonic ratio of the narrowband components,
$\bar{f}_q = q\bar{f}_p$	is the mean frequency of the higher frequency component in Hz
σ_ζ	is the standard deviation of the Gaussian statistic ζ_n in Hz
σ_ξ	is the standard deviation of the random statistic ξ_n in Hz,
σ_η	is the standard deviation of the random-noise statistic η_n,
α	is ratio of the peak-frequency fluctuation of ν_n to σ_ζ,
τ	is frequency-fluctuation smoothing time in seconds,
r_q	is the q-channel signal-to-noise rms ratio in dB/Hz, and
$\rho = r_p - r_q$	is the difference in the p-channel, and q-channel signal-to-noise rms ratios in dB/Hz.

Random Signal Generation

The first step in signal simulation is to generate two random sequences of independent samples η_n and ξ_n. The statistic η_n is a zero-mean Gaussian statistic with a standard deviation of σ_η. This statistic is used as the broadband noise signal in Eq. (9), and its power spectral density is $2\sigma_\eta^2/f_s$. The statistic ξ_n is a zero-mean random statistic with a standard deviation of σ_ξ. This statistic is used as the kernal signal in the generation of the random phase functions for the harmonically related components in Eq. (9). The method of generating the two random sequences are left to the user.

Program Algorithms

The following algorithm formulations simulate the discrete signal given in Eq. (9):

$$a_q = \frac{2}{\sqrt{f_s}} \sigma_n 10^{r_q/20} \tag{10}$$

$$a_p = a_q 10^{\rho/20} = \frac{2}{\sqrt{f_s}} \sigma_\eta 10^{r_p/20} \tag{11}$$

$$\beta = \frac{f_s \tau - 1}{f_s \tau + 1} \tag{12}$$

$$\zeta_n = \beta \zeta_{n-1} + \sqrt{1 - \beta^2} \frac{\sigma_\zeta}{\sigma_\xi} \xi_n \tag{13}$$

for $n = 0, 1, 2, 3, \ldots$

$$\nu_n = \alpha \sigma_\zeta \operatorname{Frac}\{\zeta_n / \alpha \sigma_\zeta\} \tag{14}$$

where Frac{· · ·} implies the decimal fractional part of the argument while retaining the sign of the argument.

$$Z_{q,n} = \operatorname{Frac}\{Z_{q,n-1} + (\bar{f}_q + \nu_n)/f_s\} \qquad (Z_{q,-1} = 0) \tag{15}$$

$$Z_{p,n} = \operatorname{Frac}\{Z_{p,n-1} + (\bar{f}_q + \nu_n)/qf_s\} \qquad (Z_{p,-1} = 0) \tag{16}$$

$$s_n = a_p \sin(2\pi Z_{p,n}) + a_q \sin(2\pi Z_{q,n}) + \eta_n . \tag{17}$$

EXAMPLES

To exemplify the use of the simulator, the following design parameters are selected to demonstrate the performance characteristics.

f_s = 256 Hz sample rate,
q = 1.75 harmonic ratio,
$\bar{f}_q$ = 21 Hz ($\bar{f}_p$ = 12 Hz) ,
σ_ζ = 0.125 Hz,
α = 3, and
τ = 36 s (effective smoothing time).

These parameters imply that the upper signal frequency variation is bounded between 21 ± 0.375 Hz, and the e-folding time of the fluctuating-frequency component is 18 s. (The e-folding time is the time that the autocorrelation of the fluctuating-frequency component is equal to $1/e$.) For harmonically related signals, the instantaneous frequency of the upper signal is precisely q times the instantaneous frequency of the lower signal.

Spectrogram of Example Signal Simulations

Harmonically related signals using the above parameters are generated along with a random noise signal. The signal-to-noise spectral density ratio of both signal components was set to 8 dB/Hz. A spectrogram of the resulting signal is shown in Fig. 1. The resolution of the spectrogram in the example is 1/16 Hz. The two harmonically related signal components are located at center frequencies of 12 and 21 Hz respectively. Superimposed on the spectrogram for comparison is a plot of the actual fluctuating frequency ν_n given by Eq. (14). The scale of this plot was adjusted to closely approximate that of the upper-frequency signal and placed over the spectrogram plot.

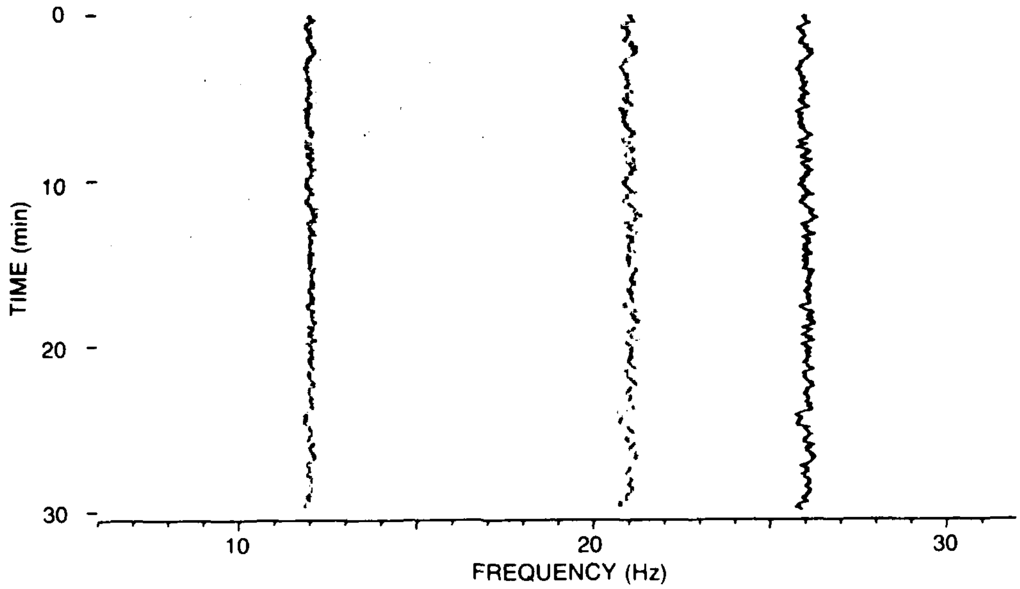

Fig. 1 – Spectrogram of simulated harmonic signals in broadband noise. The mean frequencies of the signals are 12 and 21 Hz (q = 1.75), and the signal-to-noise ratio of each signal is 8 dB/Hz. The resolution of the spectrogram is 1/16 Hz. Superimposed on the spectrogram for comparison (at approximately 26 Hz) is a plot of the actual fluctuating frequency components ν. Parameters of the signal are given in the accompanying text.

Phase Characteristics of the Simulated Signals

The phase characteristics of the harmonically related signals are obtained by using Hanning-windowed sectionalized Fourier transforms (SFTs) to filter, baseband, and decimate the two narrowband digital signals. The SFT sizes (integration times) for the upper and lower frequency signals are 2 s and 3.5 s, respectively, to contain the spectrum of the signals within the main filter passbands of 0.75 Hz and 3/7 Hz respectively [3]. The signal-to-noise power spectral density ratio in this case is 30 dB /Hz. The amplitude and phase characteristics at the filtered outputs over a 10-min. interval are shown in Fig. 2.

The top two graphs (Fig. 2) are the plots of the amplitude and phase (harmonically transformed) of the lower-frequency signal, and the middle two graphs are similar plots for the upper-frequency

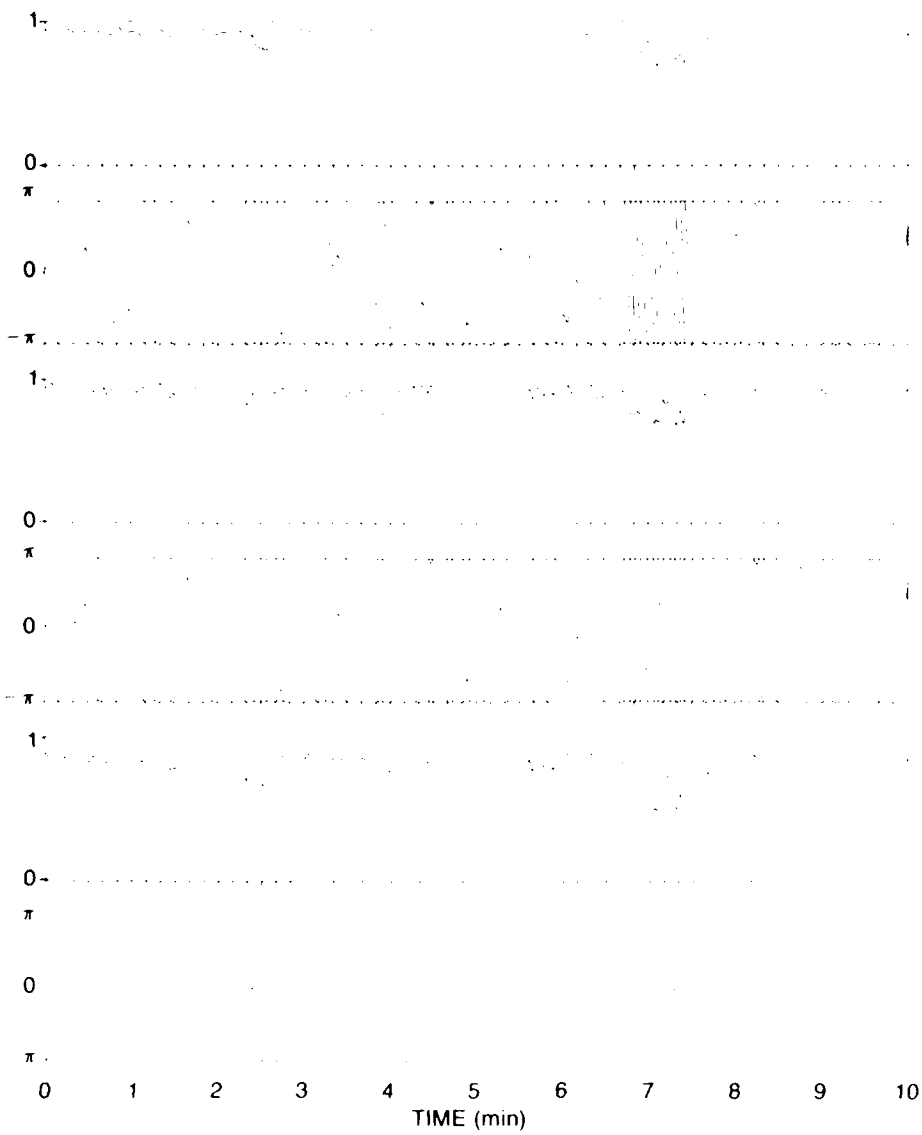

Fig. 2 Amplitude and phase plots of simulated harmonic signals showing the coherent nature of harmonically related signals. The signal-to-noise ratio of the two signals is 30 dB/Hz, and the signals have been processed through proportional SFTs whose bandwidths encompass the spectrum of the signals. The top two graphs are temporal plots of the amplitude and phase of the lower-frequency signal, and the middle two graphs are similar plots of the upper-frequency signal. The bottom two graphs show the amplitude product and the phase difference of the two signals. Harmonic correlation of the signals is the integration of the amplitude product times the cosine of the phase difference over a selected time interval.

signal. The observed amplitude fluctuations are the result of the nonuniform filter characteristics (scallop effect) of the SFT process as the instantaneous frequencies vary over time. The similarity of the harmonically related signals is quite evident. The bottom two graphs are plots of the product of the two amplitudes and the phase difference between the two signals. This is of interest since the harmonic correlation between the two signals is the integration of the amplitude product times the cosine of the phase difference over a given smoothing time. Because the phase difference is essentially zero, the harmonic correlation is effectively equal to the integral of the product of the amplitudes as expected for harmonically related narrowband signals. (The noise in the two separated filter bands is for all practical purposes uncorrelated).

To demonstrate the influence of the fluctuating-frequency components ν_n on harmonic correlation, the above experiment was repeated by using independent and uncorrelated fluctuating-frequency components for the two narrowband signals. The mean frequencies and the standard deviations of the fluctuating- signal components remain unchanged. Figure 3 shows the results.

The format of the plots in Fig. 3 is the same as for Fig. 2. In this case, however, there is little similarity between the amplitudes and phases of the two signals. In particular, the phase difference between the two signals (bottom graph) is radically variable over time. Thus, over a reasonable time increment, the integral of the amplitude product and the cosine of the phase difference can be expected to be near zero, resulting in a low (or zero) correlation. This is to be expected for narrowband signals that are not harmonically related.

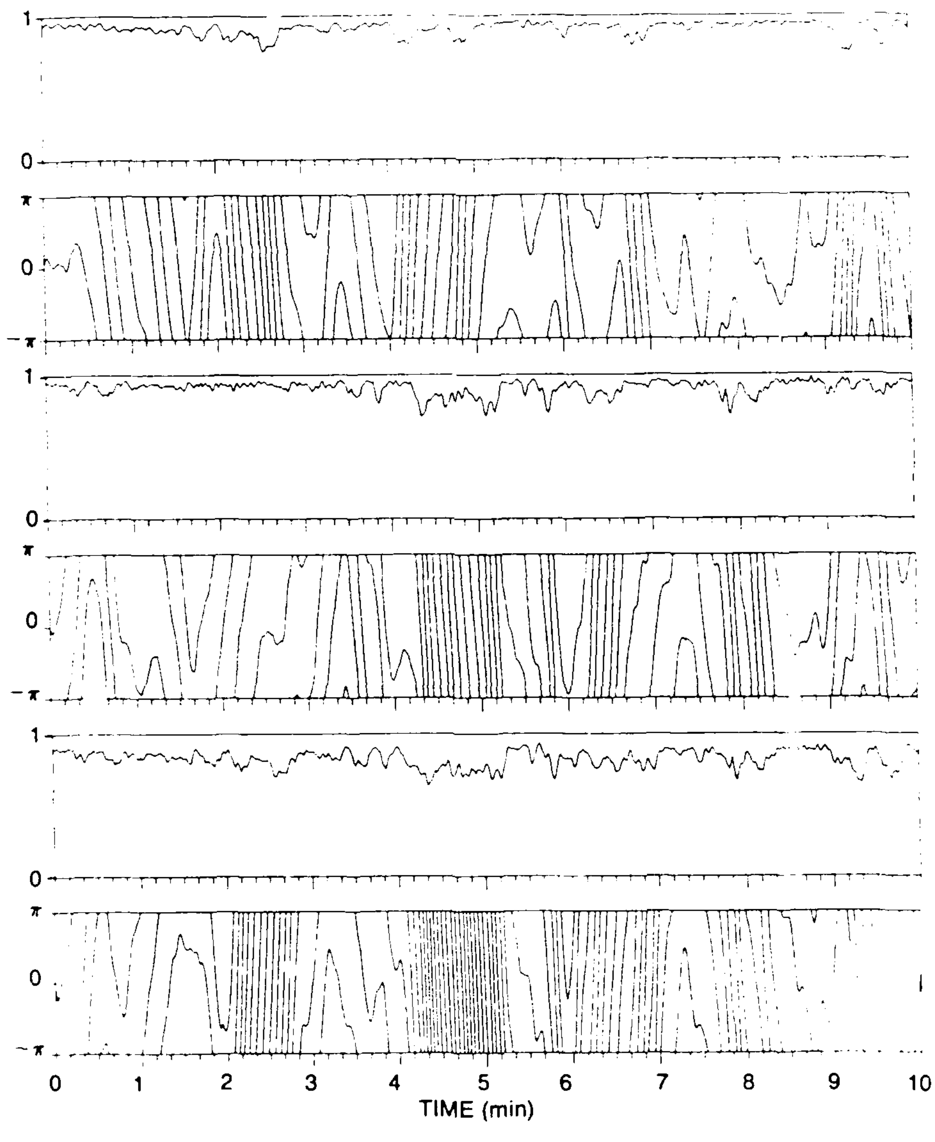

Fig. 3 - Amplitude and phase plots of simulated signals illustrating the incoherent nature of the signals when their fluctuating frequency components are independent and uncorrelated. The format of the illustration and the signal parameters are the same as in Fig. 2 with the one exception that their random fluctuating frequency components are independent. The phase difference between the signals is seen to vary radically over time, resulting in a low (or zero) correlation over relatively long integration intervals (greater than 1 min).

Instantaneous Frequency Distribution

The fluctuating frequency ν and its distribution (or probability density) are given in Eq. (3). To verify the distribution, approximately 4 million independent samples of ν were accumulated, and the distribution of ν/σ_ζ was computed. The results are shown in Fig. 4.

The data in Fig. 4 are plotted as a function of the normalized variable ν/σ_ζ for $\alpha = 3$. The continuous curve is the theoretical probability density given by Eq. (3b). The dots show the results computed from the experimental data. Although these results (computed from the large sample size) closely approximate the theoretical statistical distribution, it can be expected that results for a small sample size (such as that realized over a period of 30 min) will deviate significantly from the theoretical predictions.

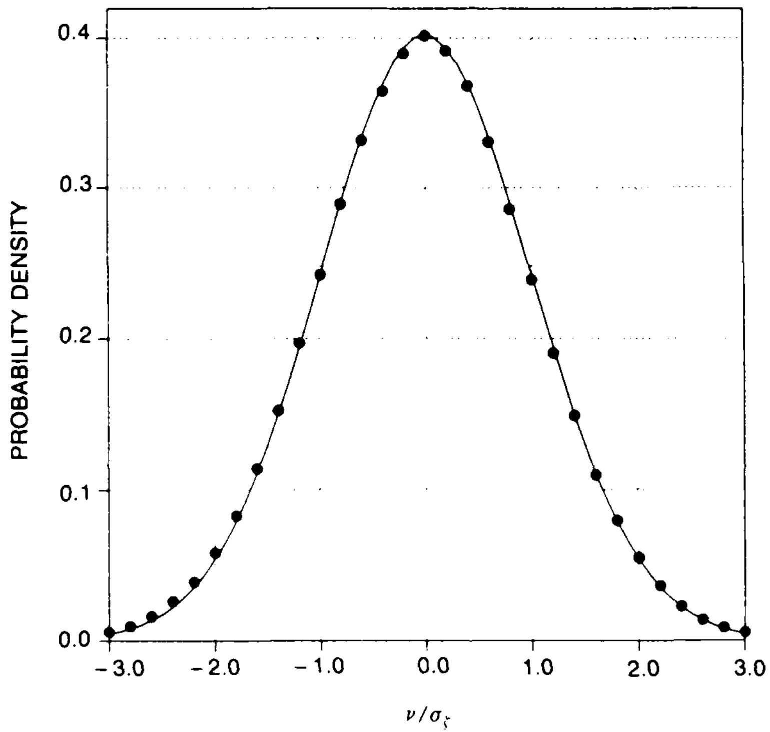

Fig. 4 — Normalized instantaneous frequency distribution of the simulated fluctuating-frequency component for $\alpha = 3$. The continuous curve is the theoretical probability density, and the solid dots are the experimental results computed from approximately 4-million independent samples of the fluctuating frequency function ν. For a small sample size, the experimental distribution can deviate significantly from the theoretical curve.

CONCLUSIONS

1. A relatively simple algorithm is formulated to simulate harmonically related narrowband signals with random frequency fluctuations in discrete format on a digital computer.
2. Signal parameters are incorporated into the algorithm to select the signal mean frequencies and to control both the spectral bounds and the autocorrelation (or power spectral density) of the signal fluctuations.
3. Examples, using the algorithm, demonstrate the performance of the signal simulator and its conformance with theoretical predictions. The results are shown in Figs. 1 through 4.

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